

ERRATUM TO *COCOMPACTLY CUBULATED GRAPH MANIFOLDS*

MARK F. HAGEN AND PIOTR PRZTYCKI

The following lemma is Lemma 4.7 of [HP15]. In the proof of part (2), we incorrectly invoked [CS11, Prop. 2.6]. Here we correct the proof, emphasizing that the statement is unchanged. The current proof is largely a re-writing of the proof of [HJP15, Lem. 6.2].

Lemma. *Consider the product of the free cyclic and a finitely generated non-abelian free group $H = \mathbb{Z} \times \mathbb{F}$. Suppose that H acts freely and cocompactly on a CAT(0) cube complex \mathcal{V} . Then the following hold:*

- (1) *The essential core \mathcal{V}^{ess} of \mathcal{V} is a product $\mathcal{V}_a \times \mathcal{V}_b$, where $\mathcal{V}_a, \mathcal{V}_b$ are unbounded.*
- (2) *The group H has a finite-index subgroup $H' = H_a \times H_b$ that preserves the above decomposition, where H_a acts trivially on \mathcal{V}_b and H_b acts trivially on \mathcal{V}_a .*
- (3) *We have $H_a = \mathbb{Z} \cap H'$ and the group H_b embeds as a finite-index subgroup of the free group $H/\mathbb{Z} \cong \mathbb{F}$ under the natural quotient.*

Proof. Since H is a direct product with infinite factors, no element is rank-one in the action on \mathcal{V}^{ess} . Corollary 6.4(iii) of [CS11] yields a nontrivial cubical product decomposition of \mathcal{V}^{ess} such that each factor has some $h \in H$ respecting the decomposition and acting on that factor as a rank-one isometry. This proves (1). By [CS11, Prop 2.6], there is a finite index subgroup $H' \leq H$ respecting this decomposition. Let \mathcal{V}_a be a factor on which $H_a = H' \cap \mathbb{Z}$ acts freely. Combine all other factors into \mathcal{V}_b , so $\mathcal{V} = \mathcal{V}_a \times \mathcal{V}_b$. Let $h_a \in H'$ act as a rank-one isometry on \mathcal{V}_a .

The generator z of H_a acts on \mathcal{V}_a as a rank-one isometry. Otherwise, no axis of h_a would be parallel to an axis of z , so that $\langle h_a, z \rangle \cong \mathbb{Z}^2$, and this subgroup acts properly on \mathcal{V}_a , contradicting that h_a is rank-one.

Consider $\text{Min}(z) = \mathbb{R} \times Y \subset \mathcal{V}_a$, where \mathbb{R} is an axis of z . Since z is central in H' , we have an induced action of H' on $\mathbb{R} \times Y$ respecting this decomposition. Since z is rank-one, Y does not contain a geodesic ray, and hence is bounded. Consequently, Y contains a fixed point of the action of H' , whence \mathcal{V}_a contains an H' -invariant line l .

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Let $\rho : H' \rightarrow \text{Isom}(l)$ be the induced map and note that $\rho(H')$ does not switch the ends of l . Since \mathcal{V}_a is a cube complex, the translation lengths on l are discrete. Thus $\rho(H')$ can be identified with the integers, containing $\rho(H_a)$ as a finite index subgroup. Let $H_b = \ker(\rho)$. Since $H_b \cap H_a = \{1\}$, the subgroup H_b embeds into $H/\mathbb{Z} = \mathbb{F}$; its image has finite index since $H' \leq H$ has finite index. Thus H_b is free, of rank ≥ 2 . Replace H' with its finite-index subgroup $H_a \times H_b$.

Choose $a \in l$. Since H_b fixes a , it acts properly on $\{a\} \times \mathcal{V}_b \subset \mathcal{V}^{\text{ess}}$, so it acts properly on \mathcal{V}_b . By hypothesis, there is a compact set $K \subset \mathcal{V}^{\text{ess}}$ such that $H'K = \mathcal{V}^{\text{ess}}$. Since H_a acts properly on \mathcal{V}_a , there are finitely many $h_a \in H_a$ for which there exists $h_b \in H_b$ so that $h_a h_b K \cap (\{a\} \times \mathcal{V}_b) \neq \emptyset$. Hence H_b acts cocompactly on $\{a\} \times \mathcal{V}_b$, and thus on \mathcal{V}_b , so any orbit map $H_b \rightarrow \mathcal{V}_b$ is a quasi-isometry.

Let $h, h' \in H_b$ be non-commuting. The sets $\text{Min}(h), \text{Min}(h') \subset \mathcal{V}_b$ have the form $\mathbb{R} \times U, \mathbb{R} \times U'$, where U, U' are bounded (since \mathcal{V}_b is quasi-isometric to a tree). Let R be large enough so that the neighbourhoods $\mathcal{N}_R(\text{Min}(h))$ and $\mathcal{N}_R(\text{Min}(h'))$ intersect. Since z commutes with h, h' , it preserves both $\text{Min}(h)$ and $\text{Min}(h')$. Thus H_a preserves $\mathcal{N}_R(\text{Min}(h)) \cap \mathcal{N}_R(\text{Min}(h'))$, which is bounded. Hence H_a fixes some $b \in \mathcal{V}_b$, so $H_b b$ is fixed by H_a . Thus H_a moves each $v \in \mathcal{V}_b$ a uniformly bounded distance.

Above, we obtained an H_a -invariant fiber $\mathcal{V}_a \times \{b\} \subset \mathcal{V}^{\text{ess}}$. As before, because $H_a \times H_b$ acts cocompactly on \mathcal{V}^{ess} , and H_b acts properly on \mathcal{V}_b , we have that H_a acts cocompactly on \mathcal{V}_a , so $l \hookrightarrow \mathcal{V}_a$ is a quasi-isometry. Since H_b fixes l , we have that H_b moves each point of \mathcal{V}_a a uniformly bounded distance. After replacing H_a and H_b (and hence H') with finite index subgroups, H_a acts trivially on \mathcal{V}_b and H_b acts trivially on \mathcal{V}_a . This proves (2). Along the way we also established (3). \square

REFERENCES

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DEPT. OF PURE MATHS AND MATH. STAT., UNIVERSITY OF CAMBRIDGE,
CAMBRIDGE, UK

E-mail address: markfhagen@gmail.com

DEPT. OF MATH. AND STAT., MCGILL UNIVERSITY, MONTREAL, QUEBEC,
CANADA

E-mail address: piotr.przytycki@mcgill.ca